

Decimals

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Introduction

Decimals are a type of fraction. Usually a fraction is written in the form $\frac{\text{numerator}}{\text{denominator}}$. Decimals are fractions in which the denominator is a power of 10, that is 10, 100, 1000 and so on, but instead of writing them in the usual form only the numerator is written down and a decimal point is used to indicate the size of the denominator. This leaflet explains this process and shows how easy it is to multiply or divide decimals by powers of ten.

Decimal fractions

Study the following fractions. In every case the denominator is a power of 10:

$$\frac{7}{10}, \quad \frac{5}{100}, \quad \frac{3}{1000}$$

In decimal form we would write

$$\frac{7}{10} = 0.7, \quad \frac{5}{100} = 0.05, \quad \frac{3}{1000} = 0.003$$

Thus the first position to the right of the decimal point indicates tenths. The second position indicates hundredths, the third indicates thousandths and so on.

A mixed number like $6\frac{3}{10}$ will consist of the whole number part on the left of the decimal point and the fractional part on the right, that is $6\frac{3}{10} = 6.3$.

A number like 0.94 can be thought of in two ways: either as $\frac{94}{100}$ or $\frac{9}{10} + \frac{4}{100}$.

Example

Convert the decimal 0.015 into a proper fraction in its simplest form.

Solution

$$\begin{aligned} 0.015 &= \frac{0}{10} + \frac{1}{100} + \frac{5}{1000} \\ &= \frac{10}{1000} + \frac{5}{1000} \\ &= \frac{15}{1000} \end{aligned}$$

This can be simplified by dividing numerator and denominator by 5 to give $\frac{3}{200}$.

Multiplying or dividing by powers of 10

To multiply and divide by powers of 10 is particularly simple. For example, to multiply 36.57 by 10 the decimal point is moved one place to the right, that is

$$36.57 \times 10 = 365.7$$

To multiply by 100 the decimal point is moved two places to the right. So

$$78.375 \times 100 = 7837.5$$

Similarly

$$0.0095 \times 1000 = 9.5$$

To divide a number by 10 the decimal point is moved one place to the left. This is equivalent to multiplying by 10^{-1} . To divide by 100 the decimal point is moved two places to the left. This is equivalent to multiplying by 10^{-2} .

For example

$$36.7 \div 10 = 3.67$$

$$5.8 \div 10 = 0.58$$

$$0.0475 \div 100 = 0.000475$$

In general, to multiply a number by 10^a the decimal point is moved a places to the right if a is a positive whole number, and a places to the left if a is a negative whole number. If necessary additional zeros are inserted to make up the required number of digits.

For example:

$$0.00743 \times 10^5 = 743.$$

$$0.056 \times 10^{-2} = 0.00056$$

Converting a fraction to a decimal

To convert a fraction into a decimal remember that $\frac{a}{b}$ means $a \div b$. Usually a calculator will be used to perform the division although in simple cases you should be able to perform the division yourself. For example

$$\frac{3}{8} = 0.375$$

When fractions are converted to decimals the result will either a) terminate, as in the previous example, or b) recur, as in $\frac{1}{3} = 0.333\dots$ which we often write as $0.\dot{3}$ and read as 'nought point 3 recurring'. Similarly $\frac{2}{9} = 0.222\dots = 0.\dot{2}$.

Furthermore $\frac{5}{33} = 0.151515\dots = 0.\dot{1}\dot{5}$.

Exercises

- Convert the following to decimals: (a) $\frac{3}{4}$, (b) $\frac{5}{6}$.
- Convert the following decimals to fractions: (a) 0.08, (b) 0.875, (c) 0.45.

Answers

- (a) 0.75, (b) $0.8\dot{3}$.
- (a) $\frac{2}{25}$, (b) $\frac{7}{8}$, (c) $\frac{9}{20}$.