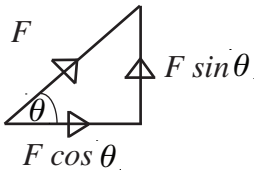
Forces acting at an angle: Resolving Forces

mc-web-mech2-6-2009

A force that acts at an angle can be split into two perpendicular components.



Newton's Second Law can be applied in each of the resolved directions.

Worked Example 1.

A computer base unit of mass 6 kg is dragged along a smooth desk. If the tension in each arm of the person dragging it is 20 N and it acts at 25° above the horizontal, what is the acceleration of the base unit and what is its normal reaction?

Solution

Figure 1 shows the forces acting on the base unit. Firstly the acceleration, a, needs to be calculated.

The resultant horizontal force is $40\cos 25°$.
Using Newton's Second Law of Motion:

$$\begin{aligned} F &= ma \\ 40\cos 25° &= 6 \times a \\ \Rightarrow a &= 6.0 \text{ m s}^{-2} (2 \text{ s.f.}) \end{aligned}$$

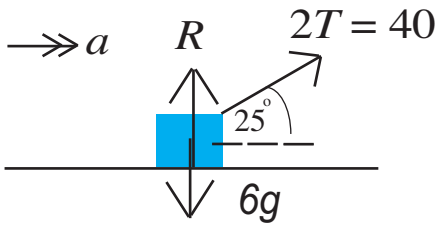


Figure 1

In order to calculate the normal reaction force, resolve vertically:

$$\begin{aligned} R + 40\sin 25° - 6 \times 9.81 &= 0 \\ \Rightarrow R = 58.56 - 16.90 &= 42 \text{ N } (2 \text{ s.f.}) \end{aligned}$$

Worked Example 2.

Two tug boats are towing a large boat, of mass 13750 kg, back to shore. Tug boat 1 is pulling with a force of $T_1 = 7500$ N at an angle of 30° north of the forward motion (see Figure 2) and tug boat 2 is pulling with a force of $T_2 = 8500$ N at an angle θ south of the forward motion. If there is a resistive motion of 1050 N opposing the eastern motion, what is the acceleration of the large boat?

Written by T. Graham, M.C. Harrison, S. Lee, C.L.Robinson

Solution

Firstly, calculate the unknown angle θ.

Resolving perpendicular to the direction of motion gives:

$$\begin{aligned} T_1 \sin 30^\circ - T_2 \sin\theta &= 0 \\ \frac{7500(\frac{1}{2})}{8500} &= \sin\theta \\ \Rightarrow \theta &= 26^\circ \text{ (2 s.f.)} \end{aligned}$$

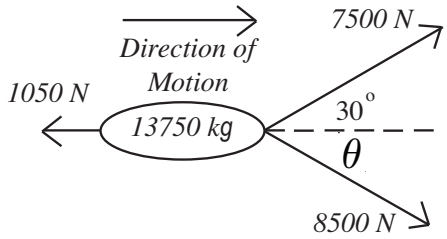


Figure 2 (View from above)

In order to calculate the acceleration, resolve in the direction of motion, which gives the resultant force as 7500 $\cos$ 30° + 8500 $\cos$ 26° − 1050.
Applying Newton's Second Law gives:

$$\begin{aligned} 7500\cos 30^\circ + 8500\cos 26^\circ - 1050 &= 13750a \\ \Rightarrow a &= 0.95 \text{ m s}^{-2} \text{ (2 s.f.)} \end{aligned}$$

Exercises

1. A computer base unit of mass 4.5 kg is dragged along a smooth desk. If the tension in each arm of the person dragging it is 16 N and acts at 22° above the horizontal, then what is the normal reaction force?
2. A computer base unit of mass 7.5 kg is dragged along a smooth desk. If the normal contact force is 23 N and the tension in the arm of the person dragging it acts at 23° to the horizontal, then what is the total tension in the person's arms?
3. Two tug boats are towing a large boat, of mass 22500 kg, back to shore. Tug boat 1 is pulling with a force of 5500 N at an angle of 35° north of the forward motion (similar to Worked Example 2) and tug boat 2 is pulling with a force of 4907.8 N at an angle 40° south of the forward motion. If the large boat is being pulled with constant velocity, and there is a resistive force to the motion, then what size is the resistive force?
4. Two tug boats are towing a large boat, of mass M kg, back to shore. Tug boat 1 is pulling with a force of T_1 N at an angle of 25° north of the forward motion (like in Worked Example 2) and tug boat 2 is pulling with a force of T_2 N at an angle of 25° south of the forward motion. If the large boat is being pulled with constant velocity, and there is a resistive force of 4000 N to the motion, then what are the magnitudes of the two forces T_1 and T_2?
5. A child on a sledge is being pulled along a horizontal path by its parent. The child and sledge have a combined mass of 20 kg and there is a normal contact force of 124.5 N. Given there is no resistance to motion and the parent pulls with a force of 125 N at an angle θ to the horizontal, then what is the angle θ?
6. A child on a sledge is being pulled along a horizontal path by its parent. The child and sledge have a combined mass of 18 kg and there is a normal contact force of 135 N. Given there is no resistance to motion and the parent pulls with a force of F N at an angle 25° to the horizontal, then what is F?

Answers (all to 2 s.f.)

1. 32 N 2. 130 N 3. 8300 N 4. $F_1 = F_2 = 2200$ N 5. 35° 6. 98 N

Written by T. Graham, M.C. Harrison, S. Lee, C.L.Robinson